**SYLLABUS: MA 502** **Don Faust**

**BIG IDEAS IN MATHEMATICS: spatial visualization, shape, and measurement**

COURSE DESCRIPTION

Historical and philosophical foundations of measurement, Euclidean and non-Euclidean geometries, analytic geometry, trigonometry, transformations, and applications.

OBJECTIVES

The teachers will develop a deeper understanding of the mathematical foundations of different geometries. The teachers will explore the role of visualization, shape and measurement in relation to understanding, describing, and solving problems encountered throughout history. The teachers will realize how such understanding can enhance their own teaching.

CONTENT FRAMEWORK

The topics will be selected from the following: Euclidean, non-Euclidean, analytical, and spherical geometries, trigonometry, measurement, graphs, and applications.

These topics could include such subtopics as the following:

Euclidean and non-Euclidean geometries: history, axioms;
analytical geometry: two and three dimensional;
spherical geometry and trigonometry: great circles, navigation;
measurement: length, area, volume, surface area;
graphs: translations, rotations, reflections, graphing calculators, software; and
applications: fundamental aspects of the science and art of applying mathematical abstractions, the fundamental nature of mathematical modeling as a tool for problem solving, examples using any of the above five topic areas.

**Discussion:**

We will attempt to gain a better understanding of the nature of mathematical work and its products. To do this, we will attempt to trace some of the strands of development which have led to its prominence today as “toolmaker for the sciences”. In this course in particular, we will focus on mathematical abstractions of space and measurement. To attempt to see more clearly the “toolmaker” character of mathematics we will try to understand with some clarity what mathematical modeling is all about and how this modeling is one of the cornerstones of the science and art of mathematical problem solving.

In fact, in making these endeavors at improving our understanding of mathematics, we will move through the thousands of years of mathematical work, beginning at least in the Upper Paleolithic of 40,000 BC. Again and again we will find the sequence: problems/querries, mathematical systems for addressing them, an increased understanding, and success! Today, and surely into many tomorrows, further iterations of this sequence will continue, leading onward through ever-improving levels of understanding. This broader perspective on the nature of mathematical work should be helpful in fact for teachers of mathematics, for this perspective will help to elucidate the pervasiveness of the concept of modeling in mathematics.

Sometimes, even though one has had a number of specific courses in mathematics, it takes a synthesis-oriented course like this one to gain a deepened understanding of why all the tools are there and also what some of the huge jobs are which remain to be done.

**Evaluation:**

There may be some submitted assignments; there probably will be some class presentations; and, there will surely be at least a short paper.

Secondly, there will be three mid-semester exams and a final exam. Only very exceptional circumstances could justify missing an exam; in these rare cases, permission must be requested in advance, and a make-up exam **(**usually oral) will be arranged for later in the semester.

The evaluation framework is as follows: (please note especially the dates, already fixed, when the three exams will take place):

| | | | |
|---|---|---|---|
| Exam 1: | Tues | 8 July | 150 points |
| Exam 2: | Tues | 15 July | 150 points |
| Exam 3: | Tues | 22 July | 150 points |
| Final Exam: | | | 300 points |
| Assignments/presentations/paper: | | | 250 points |
| | | Total: | 1000 points |

**<u>Of course, this is a graduate course: other reasonable evaluation schemes can be considered, and adopted, once we begin meeting.</u>**

**Grading:**

90-100%, A; 80-89%, B; 70-79%, C; 60-69%, D; 0-59%, F. The grading may be less stringent, but not more stringent, than this.

**Note regarding special needs:**

If you have a need for any disability-related accomodations or services, please inform the Coordinator of Disability Services Office in 405 Cohodas (227-1550). Reasonable and effective accomodations and services will be provided to students if requests are made in a timely manner, with appropriate documentation, in accordance with federal, state, and university guidelines.

Rough schedule:
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| | | |
|---|---|---|
| June | 23 | MODELING: what is "doing science" about? |
| | 24 | |
| | 25 | |
| | 26 | … and measurement |
| | 30 | … and now our main focus: geometries: models of space * |
| July | 1 | |
| | 2 | |
| | 3 | |
| | 7 | |
| | 8 | test 1 (last 50 minutes of class) |
| | 9 | |
| | 10 | |
| | 14 | |
| | 15 | test 2 (last 50 minutes of class) |
| | 16 | |
| | 17 | |
| | 21 | |
| | 22 | test 3 (last 50 minutes of class) |
| | 23 | |
| | 24 | |
| | 28 | |
| | 29 | |
| | 30 | |
| | 31 | |

* This is where our main focus for the course will be, using the NCTM monograph Deductive Systems: finite and non-Euclidean geometries by Runion and Lockwood. It will involve a really fascinating journey through Absolute Geometry (which makes no commitment about parallelism) and a number of its extensions. Certainly our work and discussions should be fun and edifying, and our perspectives about the nature of mathematics (and geometry in particular) should be broadened.

Rough list of actual topics:

MODELS:

| | |
|---|---|
| Models a: | syntax and semantics |
| Models b: | more semantics: models |
| Models c: | provability and truth |
| Models d: | consistency |

MEASUREMENT:

Functions and cardinality
How many real numbers are there?

Infinitely small numbers?

SPACE:

| | |
|---|---|
| Geometry 1: | some finite geometries |
| Geometry 1*: | the basis geometry, Absolute Geometry: postulates 1 – 4 |
| Geometry 1**: | Euclidean geometry: "one parallel" |
| Geometry 2: | Lobachevskian (hyperbolic) geometry: "many parallels" |
| Geometry 3: | Riemannian (elliptic) geometry: "no parallels" |

MATHEMATICS AS "TOOLMAKER FOR THE SCIENCES"

Broader perspective?

Ways it can help in the teaching and learning of mathematics?