SYLLABUS: **Math 484** **Don Faust**

History of mathematical thought

To gain a better understanding of the nature of mathematical work and its products we will delve into the long and fascinating history of mathematics and attempt to trace some of the strands of development which have led to its prominence today as "toolmaker for the sciences". As we move through the thousands of years of mathematical work, beginning at least in the Upper Paleolithic of 40,000 BC, again and again we will find the sequence: problems/queries, mathematical systems for addressing them, an increased understanding, and success! Today, and surely into many tomorrows, further iterations of this sequence will continue, leading onward through ever-improving levels of understanding.

Sometimes, even though one has had a number of specific courses in mathematics, it takes a synthesis-oriented course like this one to gain a deepened understanding of why all the tools are there and also what some of the huge jobs are which remain to be done.

Text: MATHEMATICAL EXPEDITIONS: Chronicles by the Explorers
By Laubenbacher and Pengelley

Evaluation:

There may be some submitted assignments; there probably will be some class presentations; and, there will surely be a paper.

Secondly, there will be two mid-semester exams and a final exam. Only very exceptional circumstances could justify missing an exam; in these rare cases, permission must be requested in advance, and a make-up exam **(**usually oral) will be arranged for later in the semester.

The evaluation framework is as follows (please note especially the dates, already fixed, when the two exams will take place):

Exam 1:	Wed	10 Oct	200 points
Exam 2:	Wed	14 Nov	200 points
Final Exam:			350 points
Submitted Assignments:			50 points
Presentations/paper:			200 points
		Total:	1000 points

Grading:

90-100%, A; 80-89%, B; 70-79%, C; 60-69%, D; 0-59%, F. The grading may be less stringent, but not more stringent, than this.

Note regarding special needs:

If you have a need for any disability-related accomodations or services, please inform the Coordinator of Disability Services Office in 405 Cohodas (227-1550).

Reasonable and effective accomodations and services will be provided to students if requests are made in a timely manner, with appropriate documentation, in accordance with federal, state, and university guidelines.

SOME FURTHER COMMENTS ON THE CONTENT OF THE COURSE: a number of areas of student learning are listed, together with some indication of what the area entails and associated students outcomes:

Problem Solving:

An understanding of the history of mathematical thought will provide enriched insight into the nature and foundations of mathematics. It will be better understood that mathematics consists of carefully constructed tools for modeling and solving a wide variety of problems in the real world. By study of the history and foundations of this modeling aspect of mathematics, students will develop problem-solving skills which are more clearly founded on a better understanding of the nature of the problem-solving process. Class discussion of major problem solving efforts, together with the course term paper, will indicate this improved understanding is taking place.

Reasoning:

The whole study of the history of mathematics will provide many opportunities for the students to make and evaluate conjectures and arguments. Their mathematical thinking skills will be enriched through improved understanding of the nature of mathematical constructions. That these reasoning skills are enhanced will be evidenced by the performance of the students in class discussions and on written exams.

Communication:

This course, as a senior-level course required of secondary mathematics education majors, provides numerous very productive opportunities for the students to improve their communication skills in presenting mathematical arguments and assessing the success of these presentations.

Connections:

This course on the history of mathematical thought provides an excellent opportunity to teach about the fascinating connections between mathematics and other disciplines as well as connections among the various subdisciplines of mathematics. Looking at the historical development of each area of mathematics, one can see the interplay between the development of mathematical structures and their use as models in a wide variety of sciences. In this course, class discussions, as well as examination questions, often touch upon such connections. Certainly a successful student in this course will come away from the course with a better understanding of the nature, utility, and indeed beauty, of such connections. And clearly, such students will have an enhanced toolkit to carry with them into their teaching of mathematics, able to regularly refer to the model-building character of mathematics and its ability to serve a broad variety of disciplines in providing structures which these disciplines can use to increase their understanding.

Number systems:

This course provides a historical treatment of the development of number concepts and number systems. This perspective will clearly be of use in teaching mathematics since it will provide a background of understanding useful in teaching about the nature of these concepts and systems, and also useful in understanding better problems students may be having in learning the concepts and systems.

Computation:

The approximative nature of mathematical structures as models is learned in this course. Such understanding is surely important for understanding the importance of estimation techniques.

Measurement:

The nature of measurement is dealt with in this course, particularly as regards the connection between mathematical structures and their applications, as for example the connection between the real number system and actual measurement in space or time.

Geometric concepts:

Geometric concepts, and their utility in visualization and problem solving, are treated regularly throughout this course since such concepts permeate the history of mathematics.

Probability and statistics:

The historical setting for the development of probability and statistics, as treated in this course on the history of mathematical thought, helps the student to see more clearly the type of problems which are best treated with such techniques. When knowledge is less than certain, as it indeed often is, the methods of probability and statistics are called for. Historical examples of this phenomenon will help the student to understand this better. As teachers, they will be able to call upon real historical examples of games of chance and situations involving uncertainty to motivate the development and use of probabilistic and statistical concepts.

Algebra:

The history of algebra is covered in this course. Historical examples help to see the great advantage of algebraic language in representing information. One way this advantage is better understood is to have the student understand by examples how abstruse and difficult much problem solving was before the advent of algebraic techniques. This course on the history of mathematics is an excellent venue for building such understanding.

Axiomatics:

In this course a careful development of the nature and role of axiomatics is given. Consideration of the historical development of deductive techniques, from Euclid right through to the present, gives the student a deeper sense of the importance of these techniques to the progress of science.

Calculus:

Of course, the calculus is considered in depth in the relevant courses. Here in this course on the history of mathematics we consider the historical development of the key concepts of the calculus, thus providing the student, and potential teacher of mathematics, with foundational understanding.

The Calculus provides, fundamentally, solutions to two problems: the development of a fruitful concept of the slope of a curve at a point on the curve; and, the development of a fruitful concept of the area bounded by a curve. After two millennia of struggle with these problems, solutions were basically constructed by the seventeenth century. But possibly what makes the Calculus so central to mathematics (and to the wide range of disciplines which make daily use of mathematics) is the great number and variety of ways which have been found in the last three hundred years, and which are continuing to be found, to make productive use of these solutions in successfully attacking many practical problems.

In first semester of the Calculus sequence, you looked carefully at the concept of the derivative (which addresses the first problem) and the concept of the integral (which addresses the second problem), and began to see how these concepts provide powerful problem-solving tools.

In the second semester of the Calculus sequence, you extended the family of functions, and the toolbox of techniques, to which these concepts of derivative and integral can be applied. Further, especially driven by the problem of the lack of certain definite integrals, you learned about the machinery of infinite series, how to represent a function $f(x)$ by a power series $s(x)$ (where, for each x, $s(x)$ is an infinite series), and how to use the Taylor Theory of these power series to generate approximations which attack this problem.

In the third semester of the Calculus sequence, you looked at the Calculus in n dimensions, usually focusing on 3 dimensions. The power and beauty of the Calculus becomes more evident in this framework of functions of more than one variable, their directional derivatives, and their gradients. For example, while a function $y = f(x)$ in

Calculus I and II graphs as a curve in the plane and we study its tangent lines, a function z = f(x,y) graphs as a surface in 3-space and we study its tangent planes.

Discrete mathematics:

Historical aspects of discrete mathematics are treated, helping to provide insight into the motivation for the development of these techniques.

Mathematics as modeling:

An enriched understanding of the nature of mathematical modeling, and its applications in the natural sciences, social sciences, business, and engineering, is a natural outgrowth of the broad exposure this course provides. For clearly, mathematical modeling is what, in the broadest sense, mathematics is all about: needs are discerned, mathematical systems are constructed to help meet those needs, and models of these systems are used as approximations in actual problem solving. The course in the history of mathematics will attempt to make clearer this essential character of mathematics as modeling.

Linear and abstract algebra:

This course on the history of mathematical thought will treat the historical aspects of linear and nonlinear algebra and of abstract algebra. Particularly emphasized will be the notion of mathematical structures in general and of structure-preserving mappings (homomorphisms, continuous maps, and so on) between them.

Diversity in the mathematics culture:

This course on the history of mathematics is particularly aimed at providing the prospective teacher with a broader view of the nature of mathematics and its place in the general culture of society and the particular culture of science. Discussion and study of under-represented groups in mathematics will take place during the course, along with a discussion of ways to improve their participation in the science of mathematics (e.g. women and minorities). Particular instances, for example, of women mathematicians like Ada Lovelace, will be discussed.

Inclusiveness and diversity:

This course in the history of mathematics will certainly increase student awareness, through its treatment of the social and scientific history of the subject, of the importance of always attempting to include all cultural groups and the importance of giving each student the chance to reach their potential in understanding the nature and function of mathematics.

Technology:

Although the importance and use of particular technologies must be covered in other courses, in this course the study of the history of the use of technologies in mathematics will occur. This will help the student to see the ever-changing nature of the use of technologies and hence will increase awareness of the importance of 'staying current' and being open throughout their careers to keeping in tune with technology changes.

Assessment:

Although this course on the history of mathematics will not treat in detail the issues and methods involving assessment, the course will provide a sense of the nature of mathematical systems and their applications. This understanding will help teachers of mathematics to be more aware of the most fundamental skills of mathematics, which in turn will aid them in providing assessments which are well-focused and do not overly emphasize the more mechanical aspects of the science. There are indeed fundamental aspects of the nature of mathematics, and our assessments need to measure whether these are being successfully communicated. Clearly this course will help teachers become more aware of these fundamental aspects of mathematics.

Doing mathematics is 'problem solving':

Indeed, doing mathematics means attacking problems: understanding the problem, designing mathematical models which approximate the context of the problem, generating a solution to the problem within the model, and assessing the degree to which the model-generated solution fits the needs of the actual problem context. This course in the history of mathematics provides a general perspective about the development of mathematical models over the last 40,000 years. This perspective, together with the particular modeling skills the student will have learned in the more focused courses taken, will provide the students with toolbox of understanding and techniques which prepares them well for the teaching of mathematics.